A Study on Optimal Motion of a Biped Locomotion Machine

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Abstract

In this paper, a calculation method of an optimal trajectory of a biped locomotion machine is proposed, which is based on the inverse kinematics and the inverse dynamics. The algorithm proposed is composed of as follows; First, the trajectory of the waist is expressed by Fourier siries, where, the bases are selected appropriately so that the periodical boundary conditions are strictly satisfied. Then, the angles of each joint are determined by the inverse kinematics. Next, using the inverse dynamics, input torques of each joint are expressed as functions of Fourier coefficients. The performance index is defined as quadratic form of input torques, and Fourier coefficients of the trajectory of the waist become its optimization parameters. Then, the motion planning problem becomes optimization problem of the trajectory of the waist whose parameters are Fourier coefficiens.

Moreover, a biped locomotion machine establishes an optimal walking by using kicking forces to the ground at the moment of switching legs. In order to include the effects of the kicking forces, additional terms that indicate impulsive forces are included as discontinuity of the velocity at the moment of switching legs.

1 Introduction

This paper deals with the optimal motion of a biped locomotion machine in the motion planning system. In this paper, the optimal motion planning is formulated and derived as the optimization problem. ([2]) The algorithm to solve this problem becomes iterative one to boundary value problem. On the point of view of kinematics of a biped locomotion machine its trajectory is periodical, and then the trajectory needs to satisfy the boundary conditions at the moment of switching legs strictly. But in order to derive such trajectory, many calculation times are required. So far, some optimization methods of the trajectories of the nonlinear multi-body systems have been proposed([3]). In some of such researches([4],[5]), the input torques are expressed by Fourier series and motion planning problem is reduced to the optimization problem with many variables. But even if we use this method, many calculation times are required in order to satisfy the constraints corresponding to the boundary conditions mentioned above.

In this paper, a calculation method of a trajectory of a biped locomotion machine is proposed, which is based on the inverse kinematics and the inverse dynamics. The algorithm proposed is composed of as follows; First, the trajectory of the waist is expressed by Fourier series, where the bases are selected appropriately so that the periodical boundary conditions are strictly satisfied. Then, the angles of each joint are determined by the the inverse kinematics. Next, using the inverse dynamics, the input torques of each joint are expressed as functions of Fourier coefficients. The performance index is defined as quadratic form of the input torques, and Fourier coefficients of the trajectory of the waist become its optimization parameters. Therefore, the motion planning problem becomes optimization problem of the trajectory of the waist whose parameters are Fourier coefficients.

On the other hand, a biped locomotion machine establishes an optimal walking by using kicking forces to the ground at the moment of switching legs. In order to include the effects of the kicking forces, many terms of Fourier series are required, but it needs many times of calculation. In this paper, it is expressed by the effects of the impulsive forces. The impulsive forces acted to the system satisfy the continuity of the trajectory, but it causes discontinuity of the velocity. Considering this fact, additional terms that indicate discontinuity of the velocity at the moment of switching legs are included to Fourier series of the trajectory of the waist.

Then, using S.Q.P. (Successive Quadratic Programming method) optimal trajectory of a biped locomotion machine is obtained.

2 Equations of Motion

We consider a biped locomotion machine composed of the main body and two legs indicated in Fig. 1. The motion of the biped locomotion machine considered here is constrained in the sagittal plane and each joint has one rotational DOF. Each leg is numbered as No.1 and 2 and leg 1 is considered as supporting leg. Each link of the leg 1 is numbered as No.1 and 2 from the supporting point to the main body and each link of the leg 2 is numbered as No.4 and 5 from the main body to the end of the leg. The main body is numbered as link 3.

We define coordinate system fixed to an inertial space whose origins are corresponded to the supporting point as $[\boldsymbol{a}_0]$. Similarly, coordinate system whose origins are on the joint i are defined as $[\boldsymbol{a}_i]$. We also define the following vectors and matrices.

$\boldsymbol{\omega}_{ii-1} = [\boldsymbol{a}_i]^T \omega_{ii-1}$: Angular velocity vector of $[\boldsymbol{a}_i]$ to $[\boldsymbol{a}_{i-1}]$.

$\boldsymbol{\omega}_{i0} = [\boldsymbol{a}_i]^T \omega_{i0}$: Angular velocity vector of $[\boldsymbol{a}_i]$ to $[\boldsymbol{a}_0]$.

A_{i0} : Rotational matrix.

We express cross product of a vector $\boldsymbol{x} = [\boldsymbol{a}]^T x$ as $\tilde{x}$ in $[\boldsymbol{a}]$ system.

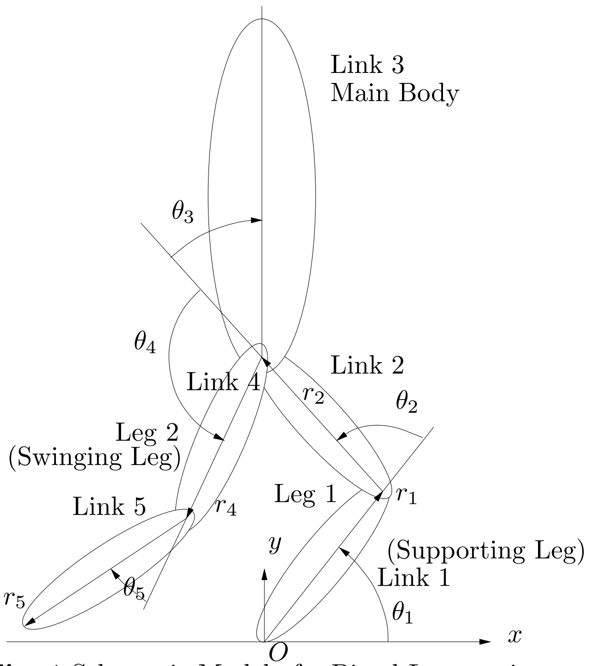

Fig. 1 Schematic Model of a Biped Locomotion

We use the vector $\hat{\omega}$ as the state variable.

$$\hat{\omega} = \left[\begin{array}{ccccc} \bar{\omega}_{10}^T & \bar{\omega}_{21}^T & \bar{\omega}_{32}^T & \bar{\omega}_{42}^T & \bar{\omega}_{54}^T \end{array} \right]^T \quad (1)$$

$$\bar{\omega}_{ij} = A_{i0}^T \omega_{ij} \quad (i = 1, 2, \cdots, 5)$$

Equations of motion concerning to generalized momenta $\hat{L}$ corresponding to state variable $\hat{\omega}$ are derived as follows;

$$\dot{\hat{L}}_1 = \hat{G}_1 \quad (2)$$

$$\dot{\hat{L}}_2 + \tilde{V}_2^T \hat{P}_2 = \hat{G}_2 + \hat{u}_2 \quad (3)$$

$$\dot{\hat{L}}_3 + \tilde{V}_3^T \hat{P}_3 = \hat{G}_3 + \hat{u}_3 \quad (4)$$

$$\dot{\hat{L}}_4 + \tilde{V}_4^T \hat{P}_4 = \hat{G}_4 + \hat{u}_4 \quad (5)$$

$$\dot{\hat{L}}_5 + \tilde{V}_5^T \hat{P}_5 = \hat{G}_5 + \hat{u}_5 \quad (6)$$

where, $\hat{u}_i$ $(i = 2, \cdots, 5)$ are input torques acted to each joint. V_i, $\hat{P}_i$, $(i = 2, \cdots, 5)$, G_j $(j = 1, \cdots, 5)$ are linear velocities and linear momenta of each link, and gravity terms, respectively.

As mentioned above, the impulsive forces caused by kicking on the ground act on a biped locomotion machine at the moment of switching legs. The variances of the angular velocities of each joint are expressed as follows;

$$\sum_j K_{ij} \Delta\omega_{jj-1} = \hat{L}_i(0) - \hat{L}_i(t_f) \quad (i = 2, \cdots, 5) \quad (7)$$

where, K_{ij} is a part of inertia matrix corresponding to angular velocity ω_{jj-1}.

And the variance of the angular velocity of the main body is expressed by using Eqs.(7) as follows;

$$\Delta\omega_{32} = K_{13}^{-1}(K_{11}\Delta\omega_{10} + K_{21}\Delta\omega_{21} + K_{42}\Delta\omega_{42} + K_{54}\Delta\omega_{54}) \quad (8)$$

$$\Delta\omega_{ij} \triangleq \omega_{ij}(0) - \omega_{ij}(t_f)$$

3 Formulation of Optimization

On the point of view of dynamical feature, a dynamical performance of a biped locomotion machine depends upon its walking trajectory because it has strong interactions with the gravitational field.

In this section, we consider an optimization of the trajectory of the biped locomotion machine.

Figure 2 shows a trajectory of a biped locomotion machine in the sagittal plane; where, S is stride.

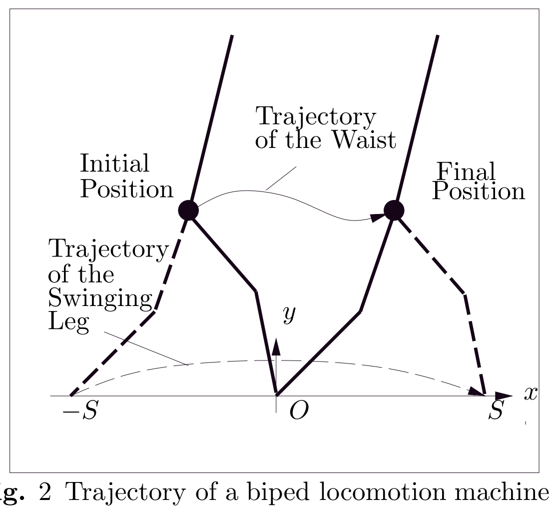

Fig. 2 Trajectory of a biped locomotion machine in the sagittal plane

3.1 Trajectories of each link

When the desired Stride S and desired Walking-Period t_f are given, the trajectory of the end of the swinging leg $\boldsymbol{z}_c = (x_c, y_c)$ is calculated as a function of time as follows;

$$x_c = x_c(t, S, t_f)\ ,\quad y_c = y_c(t, S, t_f) \tag{9}$$

On the other hand, the trajectory of the waist in the sagittal plane $\boldsymbol{z}_b = (x_b(t), y_b(t))$ needs to be expressed so that the periodical boundary conditions are strictly satisfied. Then, in this paper, trajectory of the waist is expressed using Fourier series as follows;

$$\begin{aligned} x_b(t) &= vt + \alpha_0 + \sum_{n=1}^{N} \Big\{ \alpha_{3n-2} \sin(\frac{(2n-1)\pi}{t_f} t) \\ &\quad + \alpha_{3n-1} \cos(\frac{2n\pi}{t_f} t) + \alpha_{3n} \sin(\frac{2n\pi}{t_f} t) \quad (10) \\ y_b(t) &= \beta_0 + \sum_{n=1}^{N} \Big\{ \beta_{3n-2} \sin(\frac{(2n-1)\pi}{t_f} t) \\ &\quad + \beta_{3n-1} \cos(\frac{2n\pi}{t_f} t) + \beta_{3n} \sin(\frac{2n\pi}{t_f} t) \Big\} \quad (11) \end{aligned}$$

where, $v = S/T_f$.

Note that using this expression of the trajectory, continuity of the trajectory is satisfied strictly. On the other hand, the effect of the impulsive force acted on the system is expressed as the discontinuity of the velocity at the moment of switching legs which is expressed by the terms $\sin(\frac{(2n-1)\pi}{t_f} t)$.

The trajectories of each link of the legs are calculated as functions of the trajectory of the waist and that of the end of the swinging leg as follows;

$$\left.\begin{aligned} \theta_i &= \theta_i(t, x_b, y_b) \\ \omega_{ii-1} &= \omega_{ii-1}(t, x_b, y_b, \dot{x}_b, \dot{y}_b) \\ \dot{\omega}_{ii-1} &= \dot{\omega}_{ii-1}(t, x_b, y_b, \dot{x}_b, \dot{y}_b, \ddot{x}_b, \ddot{y}_b) \\ & \qquad (i = 1, 2) \end{aligned}\right\} \tag{12}$$

$$\left.\begin{aligned} \theta_j &= \theta_j(t, x_c, y_c) \quad (j = 4, 5) \\ \omega_{42} &= \omega_{42}(t, x_c, y_c, \dot{x}_c, \dot{y}_c) \\ \omega_{54} &= \omega_{54}(t, x_c, y_c, \dot{x}_c, \dot{y}_c) \\ \dot{\omega}_{42} &= \dot{\omega}_{42}(t, x_c, y_c, \dot{x}_c, \dot{y}_c, \ddot{x}_c, \ddot{y}_c) \\ \dot{\omega}_{54} &= \dot{\omega}_{54}(t, x_c, y_c, \dot{x}_c, \dot{y}_c, \ddot{x}_c, \ddot{y}_c) \end{aligned}\right\} \tag{13}$$

The trajectories of the main body θ_3, ω_{32} are determined as follows; First, on assumption that the inclination angle of the main body to the inertial space is small enough, Eq.(2) is linearized as follows;

$$\Theta \triangleq \theta_1 + \theta_2 + \theta_3 - \pi/2 \simeq 0 \tag{14}$$

$$\begin{aligned} \dot{\omega}_{32} &= F_1(\theta_i, \omega_{ii-1}, \dot{\omega}_{ii-1})\theta_3 \\ &\quad + F_2(\theta_i, \omega_{ii-1}, \dot{\omega}_{ii-1}) \quad (i = 1, 2, 4, 5) \end{aligned} \tag{15}$$

Boundary condition

$$\theta_3(0) = \theta_3(t_f) + Ca(\theta_i(0), \theta_i(t_f)) \ (i = 1, 2) \tag{16}$$

The trajectory of the main body is given as the solution of Eq.(15) under the boundary condition Eqs.(16), (8).

3.2 Input Torque

Using the inverse dynamics, the input torque of each joint during single-supporting-phase is given as follows;

$$\hat{u}_i(\theta_j, \omega_{jj-1}, \dot{\omega}_{jj-1}) = \dot{\hat{L}}_i + \tilde{V}_i^T \hat{P}_i - \hat{G}_i \quad (i = 2, \cdots, 5)(j = 1, \cdots, 5) \tag{17}$$

On the other hand, the impulsive torques acted at each joint are expressed as follows;

$$\Delta \hat{u}_i = \hat{L}_i(0) - \hat{L}_i(t_f) \quad (i = 2, \cdots, 5) \tag{18}$$

3.3 Performance Index

In this section, Performance Index for optimization of trajectories of a biped locomotion machine is explained. Performance Index is composed of two parts. The first part evaluates the cost of the input torques of each joint in the single-supporting-phase. The other one estimates the effects of impulsive forces at the moment of switching legs. Performance Index is defined as quadratic form as follows;

$$J_L = \int_0^{t_f} \sum_{i=2}^{5} \frac{R_i}{K_{Ti}^2} \hat{u}_i^2 dt + W \sum_{i=1}^{5} \Big\{ \hat{L}_i(T_f) - \hat{L}_i(0) \Big\}^2 \tag{19}$$

where, W is a scalar weighted coefficient, and R_i, K_{Ti} are values of electrical register and torque coefficient of each actuator, respectively.

In other words, the first term of the performance index assumed to evaluate the energy consumption at the coils of electrical DC Actuators in the single-supporting-phase. The second term supposes to evaluate the impulsive one.

4 Numerical Examples

Using Motion Planning algorithm proposed, dynamical performances a biped locomotion machine are investigated.

Table 1. shows physical parameters of a biped locomotion machine, that is almost same as human parameters.

Table 1 Parameters

	Length [m]	Mass [kg]	Inertia [kgm^2]
Link 1,5	0.20	1.5	5.0×10^{-3}
Link 2,4	0.20	2.0	7.0×10^{-3}
Link 3	0.30	5.0	1.0×10^{-2}

The trajectory of the waist is expressed using Fourier series up to the 2nd mode.

A biped locomotion has some geometrical constraints of its feature as follows;

$$\|\boldsymbol{z}_b\|_2 \leq (r_1 + r_2) \qquad t \in [0, t_f] \tag{20}$$

$$\|\boldsymbol{z}_c - \boldsymbol{x}_b\|_2 \leq (r_4 + r_5) \qquad t \in [0, t_f] \tag{21}$$

$$\theta_1(\boldsymbol{z}_b) \geq 0 \qquad t \in [0, t_f] \tag{22}$$

where, $\| * \|_2$ is Euclid Norm.

Figs. 4 show stick figures of a biped locomotion machine without and with the impulsive forces. **Fig.** 5 shows the performance index selecting weight value of the performance index W as a parameter. From these figures, an optimal trajectory of a biped locomotion machine is planned with the algorithm proposed in this paper, and we may note that a biped locomotion machine has some walking patterns according to whether it use the impulsive forces actively or not.

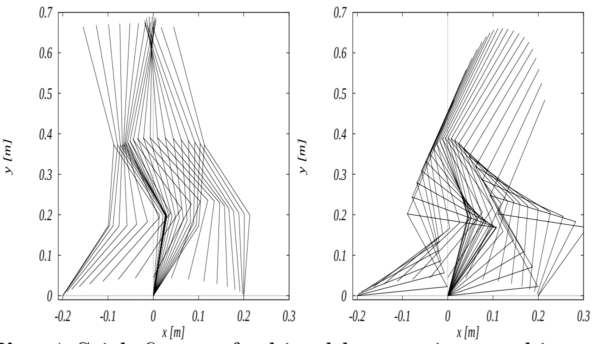

Fig. 4 Stick figure of a biped locomotion machine Left: without impulsive force, Right: with impulsive force

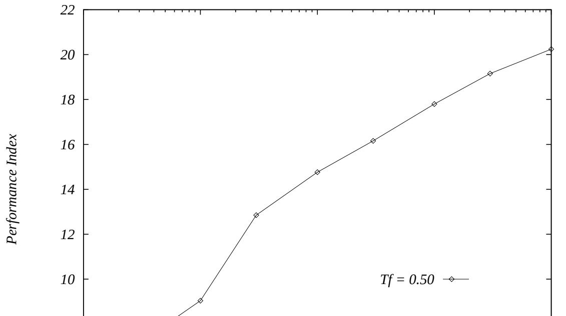

Fig. 5 Performance index (W as parameter)

5 Conclusion

A motion planning method of a biped locomotion machine is proposed. The trajectory of the waist is expressed by Fourier Series satisfying continuity conditions of the trajectory at the moment of switching legs and it also includes the terms that express discontinuity of the velocity at the moment of switching legs as the effects of the impulsive forces. Based on the inverse kinematics, the trajectories of each joint are derived. Then, using inverse dynamics, the input torques are calculated, and performance index is defined by two parts, that is, quadratic form of the input torques and the effects of impulsive forces at the moment of switching legs. For the optimization algorithm, S.Q.P. method is used. Using these formulation, an optimal trajectory of a biped locomotion machine is obtained.

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