A Study on Optimal Gait Pattern of a Quadruped Locomotion Robot *

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Abstract – *This article deals with the optimal gait pattern in terms of the proposed control system. The phase difference among the oscillators is selected as the optimization variable. The duty ratio is selected as the optimization parameter. The performance index is constructed as a quadratic form of the input torque vector at the joints. The simulated annealing (SA) method is used as the optimization algorithm and numerically derived the optimal gait patterns according to the various duty ratios.*

Keywords: Optimal gait pattern, Quadruped locomotion, Simulated annealing.

1 Introduction

Locomotion is one of the basic functions of a mobile robot. Using leg is one of the strategies for accomplishing locomotion. It allows the robot to move on rough terrain. According to the recent researches on control system for legged robots, it has been known that multi-legged locomotion robots such as quadruped, hexapod, etc. have many feasible gait patterns and it is pointed out that such robots have possibilities of emergence of appropriate gait patterns according to the variance of the environment. This article treats the optimization problem of gait patterns for a quadruped robot.

Hoyt and Taylor investigated relationship among locomotion speed, emerged gait pattern and Oxygen consumption in terms of pony [1]. The results are summarized as; each typical gait pattern emerges at the locomotion speed where Oxygen consumption is minimum, and the optimal value of Oxygen consumption for each gait pattern is almost same with each other. This implies that the pony can generate the optimal gait pattern according to the variance of the environment. One of the effective strategies to realize artificial system that has such performance is the minimization of the cost of transport. Based on such idea, a considerable amount of researches on optimal gait pattern of legged robots has been done [2]-[7].

On the other hand, in the point of view of control, the legged robots will be required which can carry out various tasks on unstructured terrain in the future. The legged robots are required to realize the real-time adaptability to a changing environment. Therefore, a considerable amount of researches on motion control system for the legged robots has been done that can adaptively generate various gait patterns and realize stable locomotion according to the variance of the environment [8]-[10].

The authors have proposed a control system for a quadruped locomotion robot using nonlinear oscillator network [11]. This control system consists of a gait pattern controller and a leg motion controller. The gait pattern controller involves nonlinear network. The leg motion controller controls the actuators mounted at the joints of the legs. The phase difference leads to the gait pattern of the robot.

This article deals with the optimal gait pattern in terms of the proposed control system. The phase difference among the oscillators is selected as the optimization variable. The locomotion speed is parameterized with the duty ratio that expresses the ratio of time period for supporting stage against the locomotion period. The duty ratio is selected as the optimization parameter. The performance index is constructed as a quadratic form of the input torque vector at the joints. The simulated annealing (SA) method is used as the optimization algorithm and numerically derived the optimal gait patterns according to the various duty ratios.

2 Model

Consider the quadruped locomotion robot shown in Fig. 1, which has four legs and a main body. Each leg is composed of two links which are connected to each other through a one degree of freedom (DOF) rotational joint. Each leg is connected to the main body through a one DOF rotational joint. Legs are enumerated from leg 1 to 4, as shown in Fig. 1. The joints of each leg are numbered as joint 1 and 2 from the main body toward the tip of the leg. We define $r_i^{(0)}$ and $\theta_i^{(0)}$ $(i = 1,\ 2,\ 3)$ as the components of position vector and Euler angle from inertial space to the coordinate system which is fixed on the main body, respectively. We also define $\theta_j^{(i)}$ as the joint angle of link j of leg i.

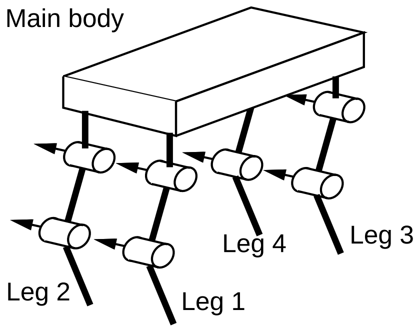

Figure 1: Schematic model of a quadruped robot

The state variable is defined as follows;

$$q^T = \left[\begin{array}{ccc} r_k^{(0)} & \theta_k^{(0)} & \theta_j^{(i)} \end{array} \right] \quad (1)$$
$$(i = 1, \cdots, 4, \quad j = 1, 2, \quad k = 1, 2, 3)$$

Equations of motion for state variable q are derived using Lagrangian formulation as follows;

$$M\ddot{q} + H(q,\ \dot{q}) = G + \sum(\tau_j^{(i)}) + \Lambda \quad (2)$$

where M is the generalized mass matrix and the term $M\ddot{q}$ expresses the inertia. $H(q,\ \dot{q})$ is the nonlinear term which includes Coriolis forces and centrifugal forces. G is the gravity term. $\tau_j^{(i)}$ is the input torque of the actuator at joint j of leg i. Λ is the reaction force from the ground at the point where the tip of the leg makes contact. We assume that there is no slip between the tips of the legs and the ground.

3 Control system

3.1 Design of the trajectories of the legs

The position of the tip of the leg where the transition from the swinging stage to the supporting stage occurs is called the anterior extreme position (AEP). Similarly, the position where the transition from the supporting stage to the swinging stage occurs is called the posterior extreme position (PEP). We determine the nominal trajectories that are expressed in the coordinate system which is fixed on the main body. First, we define the nominal PEP $\hat{r}_{eP}^{(i)}$ and the nominal AEP $\hat{r}_{eA}^{(i)}$ (Fig. 2). The index $\hat{*}$ indicates the nominal value. The trajectory for the swinging stage is a closed curve given as the nominal trajectory $\hat{r}_{eF}^{(i)}$. This curve involves the points $\hat{r}_{eA}^{(i)}$ and $\hat{r}_{eP}^{(i)}$. On the other hand, the trajectory for the supporting stage is a linear trajectory given as $\hat{r}_{eS}^{(i)}$. This linear trajectory also involves the points $\hat{r}_{eA}^{(i)}$ and $\hat{r}_{eP}^{(i)}$. The position of each leg on these trajectories is given as functions of the phase of the corresponding oscillator.

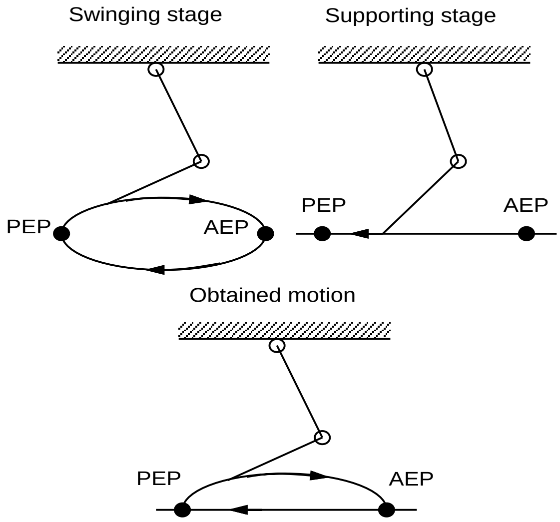

Figure 2: Trajectory of the leg

The state of the oscillator for leg i is expressed as follows;

$$z^{(i)} = \exp(j\ \phi^{(i)}) \quad (3)$$

where $z^{(i)}$ is a complex number representing the state of the oscillator, $\phi^{(i)}$ is the phase of the oscillator and j is the imaginary unit.

The nominal phases at AEP and PEP are determined as follows;

$$\hat{\phi}^{(i)} = \hat{\phi}_A^{(i)} \quad \text{at AEP}, \qquad \hat{\phi}^{(i)} = \hat{0} \quad \text{at PEP} \quad (4)$$

The nominal trajectories for swinging stage $\hat{r}_{eF}^{(i)}$ and for supporting stage $\hat{r}_{eS}^{(i)}$ are given as functions of the phase $\hat{\phi}^{(i)}$ of the oscillator and are alternatively switched at every step of AEP and PEP.

$$\hat{r}_e^{(i)}(\hat{\phi}^{(i)}) = \begin{cases} \hat{r}_{eF}^{(i)}(\hat{\phi}^{(i)}) & 0 \le \hat{\phi}^{(i)} < \hat{\phi}_A^{(i)} \\ \hat{r}_{eS}^{(i)}(\hat{\phi}^{(i)}) & \hat{\phi}_A^{(i)} \le \hat{\phi}^{(i)} < 2\pi \end{cases} \quad (5)$$

The nominal duty ratio $\hat{\beta}^{(i)}$ for leg i is defined to represent the ratio between the nominal time for the supporting stage and the period of one cycle of the nominal locomotion.

$$\hat{\beta}^{(i)} = 1 - \frac{\hat{\phi}_A^{(i)}}{2\pi} \quad (6)$$

3.2 Definition of the gait patterns

The gait patterns, which are the relationships between motions of the legs, are defined.

Each pattern is represented by a matrix of phase differences Γ_{ij} as follows;

$$\phi^{(j)} = \phi^{(i)} + \Gamma_{ij} \quad (7)$$

Because the motion of the leg is periodical, and the origin of the absolute phase is arbitrary, it is enough to consider three variables in terms of phase difference to determine the gait pattern uniquely. In this study, a set of the variables is selected to determine the gait pattern uniquely and is selected the optimization variable, as follows;

$$c = \begin{bmatrix} \Gamma_{21} \\ \Gamma_{31} \\ \Gamma_{41} \end{bmatrix} = \begin{bmatrix} \phi^{(1)} - \phi^{(4)} \\ \phi^{(2)} - \phi^{(4)} \\ \phi^{(3)} - \phi^{(4)} \end{bmatrix} \tag{8}$$

3.3 Leg motion control

The angle of joint j of leg i is derived from the geometrical relationship between the trajectory $\hat{r}_e^{(i)}(\hat{\phi}^{(i)})$ and the joint angle. $\hat{\theta}_j^{(i)}$ is written as a function of phase $\hat{\phi}^{(i)}$ as follows;

$$\hat{\theta}_j^{(i)} = \hat{\theta}_j^{(i)}(\hat{\phi}^{(i)}) \tag{9}$$

The commanded torque at each joint of the leg is obtained by using local feedback control as follows;

$$\begin{aligned} \tau_j^{(i)} &= K_{Pj}(\hat{\theta}_j^{(i)} - \theta_j^{(i)}) + K_{Dj}(\dot{\hat{\theta}}_j^{(i)} - \dot{\theta}_j^{(i)}) \quad (10) \\ & \qquad (i = 1, \cdots, 4, j = 1, 2) \end{aligned}$$

where $\tau_j^{(i)}$ is the actuator torque at joint j of leg i.

4 Optimization algorithm

At first, the state variable for optimization is defined as follows;

$$c_i = \phi^{(i)} - \phi^{(4)} \tag{11}$$

The performance index U is defined as follows;

$$U(c_i) = \frac{\displaystyle\int_{t_m}^{t_m+T} \sum_{i,j} (\tau_j^{(i)})^2 dt}{\displaystyle\int_{t_m}^{t_m+T} \dot{r}^{(0)} dt} \tag{12}$$

where,t_m is the time after when the effect of the transient motion vanishes. T is the measuring time.

The optimization of the gait pattern (i.e. phase difference among the oscillators) is implemented by using Simulated Annealing method (SA) as follows:

1. A set of phase differences among the oscillators c is selected.

2. Let the robot try to walk at that phase difference c by using the dynamic simulation and measure the performance index U.

3. Temperature Θ is determined as a function of annealing time t_a.

$$\Theta = \Theta(t_a)$$

4. Revise the phase difference from c to c'. The value of modification is subject to Gaussian distribution $P_r(v)$ as a function of temperature Θ.

$$\begin{aligned} c' &= c + \Delta c & (13) \\ \Delta c &= v \Delta t_a & (14) \\ P_r(v_i) &= \left(\frac{m}{2\pi k \Theta}\right)^{\frac{1}{2}} \exp\left(-\frac{m v_i^2}{2k\Theta}\right) & (15) \end{aligned}$$

where, m and k are constant numbers.

5. Let the robot try to walk at that phase difference c' by using the dynamic simulation and measure the performance index U'.

6. If U' is smaller than or equal to U, revision of the phase difference from c to c' is always accepted. Otherwise, revision of the phase difference from c to c' is accepted at the probability P_r subject to Gibbs distribution (eq. (16)).

$$P_r = \begin{cases} \exp(-\frac{\Delta U}{k\Theta}) & (\Delta U > 0) \\ 1 & (\Delta U \leq 0) \end{cases} \tag{16}$$

• When revision is accepted

$$c := c' = c + \Delta c, \quad U := U' = U + \Delta U \tag{17}$$

• When revision is rejected

$$c := c, \quad U := U \tag{18}$$

7. Revise the annealing time t_a for the next iteration.

8. Iterate steps 2 to 7 during the annealing period.

Using the optimization procedure, numerical study is implemented. The physical parameters used in this work are shown in the following table. These are the parameters of the hardware equipment.

5 Numerical analysis

By using the Simulated Annealing method mentioned in the previous section, numerical analyses are implemented. Table 1 shows the physical parameters of the robot.

Table 1

Body	Length	0.338	[m]
	Width	0.200	[m]
	Mass	2.03	[kg]
Legs link 1	Length	0.188	[m]
	Mass	0.918	[kg]
Legs link 2	Length	0.193	[m]
	Mass	0.595	[kg]

The nominal stride $\hat{S}^{(i)}$ is fixed as 0.07 [m]. The nominal duty ratio $\hat{\beta}^{(i)}$ is selected as the simulation parameter. Numerical analyses are implemented to investigate relations of the performance index and the optimal gait pattern to the duty ratio.

During the numerical simulation using simulated annealing method, temperature is controlled as follows;

$$\log\Theta = \log\Theta_0 + \log(e_{ini} - n_{itr}\Delta t) \tag{19}$$

where, Θ_0, e_{ini},Δt are constant numbers and n_{itr} is the iteration number.

Figure 3 shows time history of the controlled temperature and the performance. From this figure, it is found that the performance index well converged to the optimal or the sub-optimal value by controlling the temperature in simulated annealing method.

The obtained results are shown in Figures 4 and 5 for example. Figure 4 shows the performance index in terms of various nominal gait patterns. Duty ratio is selected as the variable parameter. In the figure, the plots are the case of feasible solution, and the solid lines indicate the optimal value of the performance index that is obtained through the simulated annealing method. From this figure, the optimal value of the performance index has little variation, while a considerably large variation of the duty ratio is given. And the optimal values of performance index are smaller than other nominal gait patterns. This implies that the obtained optimal gait pattern has a very good performance in terms of energy efficiency at various locomotion speeds. Figure 5 shows the normalized phase difference between the oscillator for leg 4 and the other oscillators according to the duty ratio, which leads to the optimal gait patterns. From the figure, there is a tendency that the optimal gait pattern for considerably slow-speed locomotion is similar to the trot pattern, while that for high-speed locomotion is almost the canter pattern.

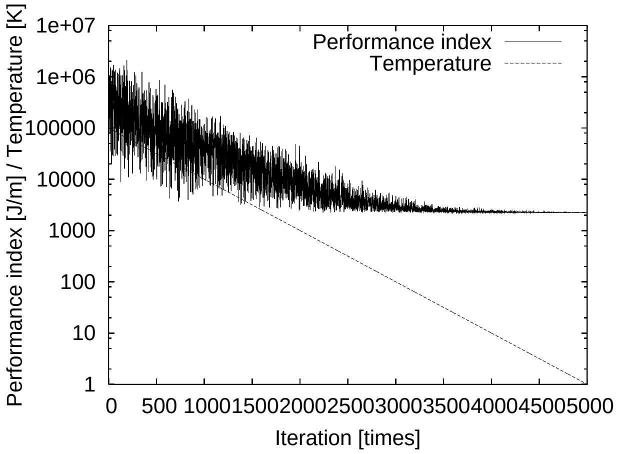

Figure 3: Controlled temperature and performance index

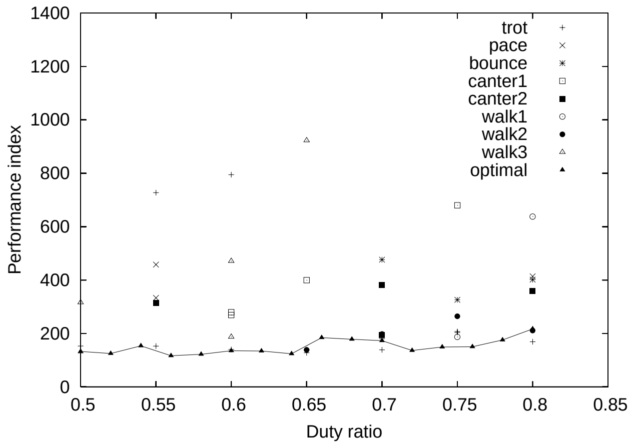

Figure 4: Performance index in terms of various gait patterns

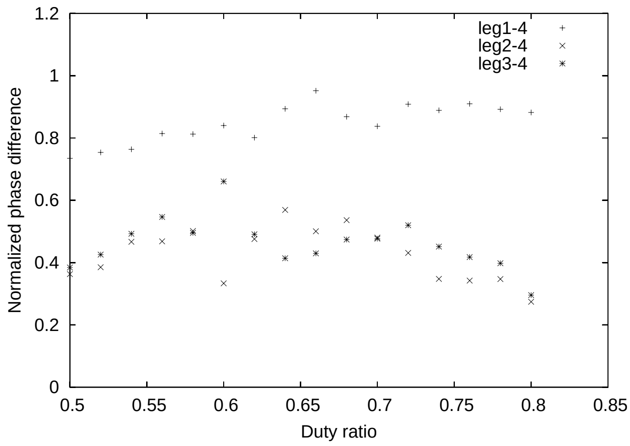

Figure 5: Obtained gait pattern

6 Conclusions

An optimization of the gait patterns for a quadruped locomotion robot is numerically implemented by using simulated annealing method. The obtained results are summarized as follows: The optimal value of the performance index has little variation while a considerably large variation of the duty ratio is given. And the optimal values of performance index are smaller than other nominal gait patterns. Under a certain condition, obtained gait patterns are the trot pattern for slow-speed locomotion and the canter pattern for high-speed one.

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